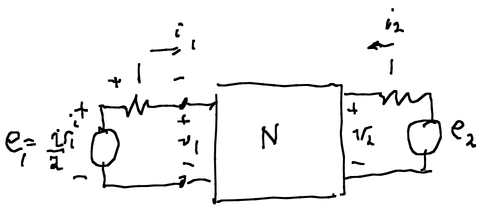
ENEE 610
09/24/20

Note Title 9/19/2020

Homework 3
VLSI tutorial
Good topic paper: Optimal area allocation for resistors and capacitors in continuos - time monolithic filters, H. Fei, R. L. Geige and D. Chen, ISCAS proceedings, 2004, p. I-865 -I-868

Existence of S for passive circuits, L2 functions, Circulator, multiplication of S's; companion matrix forms, Bilateral Laplace transform

$V^r(s) = S(s)V^i(s)$ $\quad 2V^i = V + I$
$\quad 2V^r = V - I$



if passive N apply $v^i(t)$ $p(t) = v^T(t) i(t)$ if $v^i(t)$ is square integrable L_2

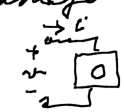
if $\int_{-\infty}^{t} v^{iT}(\tau) v^i(\tau)\, d\tau$ is finite

$= \int_{-\infty}^{t} v^T(\tau) v(\tau) + i^T(\tau) i(\tau) + \underbrace{2 v^T(\tau) i(\tau)}_{\text{non-negative if passive}}$

even as $t \to \infty$

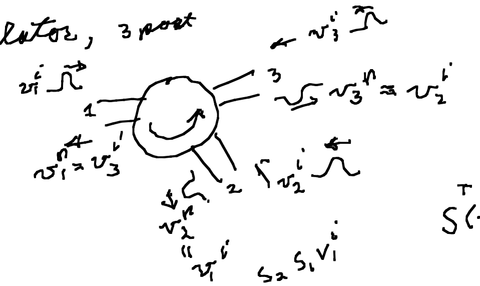
$\Rightarrow$ if N is passive & we apply L_2 v^i
then $S(j\omega)$ exists
$S(j\omega)$ is rational in ω & then $S(s)$ exists as $S(j\omega)\Big|_{\omega = s/j}$

$S(s)$ is bounded real
if rational all coefficients are real
For example no Z or Y for a ideal transformers.
But nullators are passive but no $S(s)$.



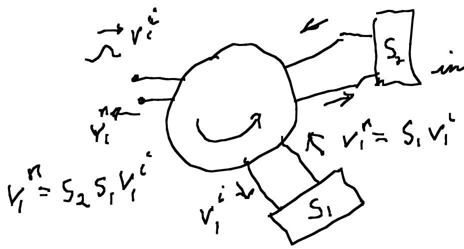
$v = 0$
$i = 0$ $\quad p(t) = 0$

Circulator, 3 port



$$S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow V^n = S V^i \quad \begin{bmatrix} v_1^i \\ v_2^i \\ v_3^i \end{bmatrix}$$

$$S^T(-s)\, S(s) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 1_3$$



incident on 2 $S_1 V_1^i$

$\Rightarrow S_{in}(s) = S_2(s)\, S_1(s)$

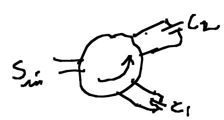
S_{in} is BR ← bounded real & rational

if S_1 & S_2 are

useful for synthesis

Ex: $S_{in}(s) = \left(\frac{1-C_1 s}{1+C_1 s}\right)\left(\frac{1-C_2 s}{1+C_2 s}\right)$ choose $S_1(s) = \frac{1-C_1 s}{1+C_1 s} \Rightarrow C_1$

$S_2 = \frac{1-C_2 s}{1+C_2 s} \Rightarrow C_2$ $\quad y = cs$, $S(s) = \frac{1-y}{1+y}$



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s: $s = \sigma + j\omega$, use in the full complex plane, take out singularities ($s = -1/C_1, -1/C_2$ above)

s = Laplace transform variable, needs a region of convergence of integral, for S_1 in $\mathrm{Re}\, s > -1/C_1$

$s = d/dt$ operator, needs operator domain of functions

s: in e^{st} Ex: $\frac{1-cs}{1+cs} = \frac{V^n}{V^i} \Rightarrow (1-cs)V^i = (1+cs)V^n$

$v^i - \frac{d}{dt} c v^i = v^n + c\frac{d}{dt}v^n \Rightarrow$ Let $v^i = V^i e^{st}$, $v^n_{c+1} = V^n e^{st}$

$V^i e^{st} - cV^i(s e^{st}) = V^n e^{st} + cV^n(s e^{st})$

$(1-cs)V^i = (1+cs)V^n$ here s is in e^{st}

Bilateral Laplace transform $\Rightarrow$ Fourier transform if $s = j\omega$; exists for $f(t) \in L_2$

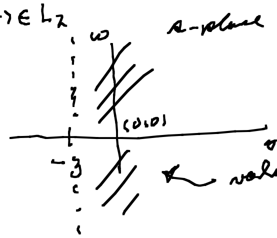
$\mathcal{L}[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-st}\, dt$ Ex: $f(t) = e^{-3t} 1(t)$, $1(t) = \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t < 0 \end{cases}$

$\mathcal{L}[e^{-3t} 1(t)] = \int_0^{\infty} e^{-3t} e^{-st}\, dt = \int_0^{\infty} e^{-(s+3)t}\, dt = \frac{1}{-(s+3)} e^{-(s+3)t}\Big|_{t=0}^{\infty}$

$= \frac{1}{s+3} + \frac{1}{-(s+3)} e^{-(s+3)\infty}\}$ < 0 for a finite value

$Re(s+3) > 0$, $Re\, s > -3$

s-plane



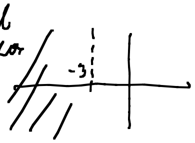
$\leftarrow$ valid region for $\mathcal{L}[e^{-3t} 1(t)]$

$\mathcal{L}[-e^{-3t} 1(-t)] = \int_{-\infty}^{0} -e^{-(3+s)t}\, dt = \frac{+1}{s+3} e^{-(3+s)t}\Big|_{t=-\infty}^{0} = \frac{+1}{s+3} + \frac{1}{s+3} \underbrace{e^{-(s+3)(-\infty)}}_{\Rightarrow 0 \text{ if } Re(s+3) < 0}$

$Re\, s < -3$

$= \frac{1}{s+3}$, $Re\, s < 0$

valid region for $\mathcal{L}[-e^{-3t} 1(-t)]$



$\therefore$ Given $F(s)$ need to know the region of converge to invert it to find $f(t)$. for $F(s) = \mathcal{L}[f(t)]$

ODE via state & companion matrix

$$\frac{V_{out}}{V_{in}}(s) = \frac{1}{s^{\delta} + d_{\delta-1} s^{\delta-1} + \dots d_1 s + d_0}$$

δ = degree $\quad V_{out} = v_0$

$$v_{in}(t) = (s^{\delta} + d_{\delta-1} s^{\delta-1} + \dots + d_1 s + d_0) v_0(t)$$

$$\dot{x}_1 = \dot{v}_0 = x_2$$

$$\dot{x}_2 = x_3 \quad \dots \quad \dot{x}_{\delta-1} = x_{\delta}$$

but $s^{\delta} v_0 = \overset{\dots\delta}{x_1} = + V_{in} - (d_{\delta-1} x_{\delta} \dots$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdot & \cdots & \cdot & 1 \\ \hline -d_0 & -d_1 & \cdots & & & -d_{\delta-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{\delta} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} V_{in}$$

state eqs. for $\frac{V_0}{V_{in}} = \frac{1}{D(s)}$

$\uparrow$ polynomial

$$v_0 = [1, 0, \dots \quad 0]$$

allows design via OTA's, C's.